Oscillations

Periodic Motion : Repeats itself after regular interval of time.

Oscillatory Motion / Harmonic Motion

→ It is a part of periodic motion
→ To and fro (back and forth or up and down) about fixed point after regular interval of time ☆

→ Fixed point about which body oscillates is called mean position or equilibrium position

$$F_{net}, a_{net} = 0$$

All oscillatory motion are Periodic but all periodic motion are not oscillatory

Date

Condition for any oscillatory motion

$F = -Kx^n$ ($n = 1, 3, 5, 7 \ldots$ any odd integer)

F = Restoring force
K = constant
* Always acts towards mean position

Simple Harmonic Motion

$F = -Kx$ ($n=1$)

x ↳ displacement from mean position

→ F depends on x ($\uparrow x = \uparrow F$)

Date

Basic Terms Related to S.H.M

1] Amplitude (A)
Extreme position covered by mass

2] Time Period (T)
Time for 1 complete oscillation

3) Frequency and Angular frequency

$f = T^{-1}$ or $\frac{1}{T}$ (in s^{-1} or heartz)

$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$ in rad/s

4) Restoring force

$F = -Kx$

$\boxed{K = m\omega^2}$ $\boxed{F = -m\omega^2 x}$

m = mass of particle

Date

5) Acceleration (a)

$$\vec{a} = \frac{\vec{f}}{m} = \frac{-kx}{m}$$

$$\boxed{a = -\omega^2 x}$$

$$\boxed{|a_{max}| = \omega^2 A}$$

6) Velocity (v)

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v = \pm \omega \sqrt{A^2 - x^2}$$

$$\boxed{|v_{max}| = \omega A}$$

The distance covered by a particle undergoing SHM in one time period is $= 4A$

Date

Equation for S.H.M

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\boxed{x = A\sin(\omega t + \phi)}$$

(if t=0, $\phi \rightarrow$ initial phase

$0 \leq \phi < 2\pi$

x = disp. from mean position

A = Amplitude

$(\omega t + \phi)$ = phase

$x = A\cos(\omega t + \phi)$

$x = A\sin\omega t + B\cos\omega t$ } S.H.M

$x = A + B\sin\omega t$

Date

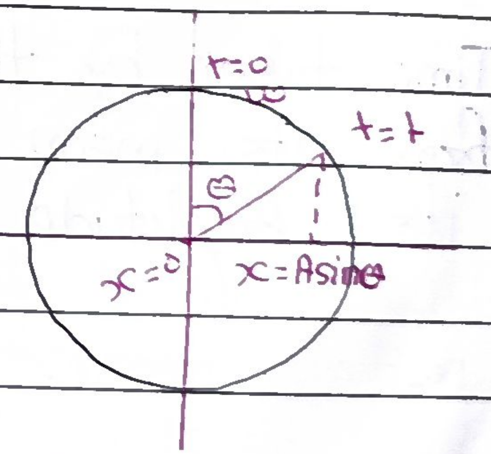
S.H.M as projection of uniform circular motion

- Uniform circular motion is not a S.H.M
→ Projection on two diameters is doing S.H.M but particle does no

- Projection on horizontal diameter

Particle - circular motion

Angular velocity ω

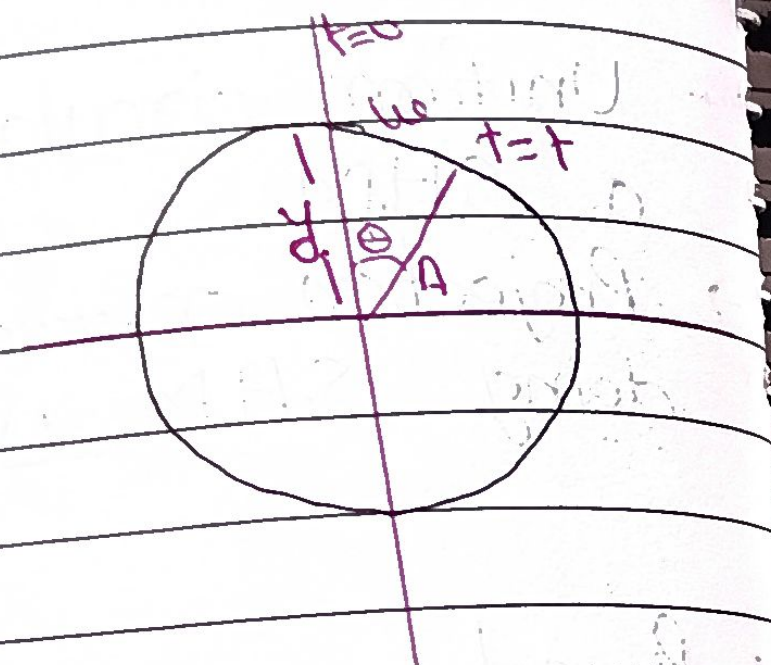


$x = A\sin\theta$

$\boxed{x = A\sin\omega t}$ $(\theta = \omega t)$

Date

vertical diameter



ken by the particle to go
is mean position to half of
Amplitude is $\left(t = \frac{T}{12}\right)$

Date

Energy in S.H.M

- Kinetic energy

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) \quad (\because v^2 = \omega^2(A^2 - x^2))$$

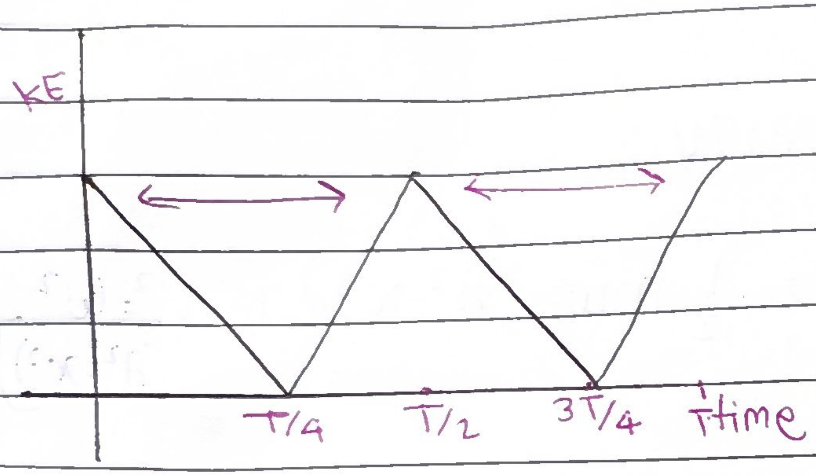
$$\boxed{KE = \frac{1}{2}k(A^2 - x^2)} \quad (k = m\omega^2)$$

Extreme
$x = \pm A$
$KE = 0$
(minimum)

mean Position
$x = 0$
$KE = \frac{1}{2}kA^2$
(maximum)

Date

GRAPH



- KE complete two cycle in an oscillation
- Frequency of KE is double of S.H.M $(f' = 2f)$

Date

KE v/s time

$(v = A\omega\cos\omega t)$

$$KE = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m A^2 \omega^2 \cos^2 \omega t$$

$$= \frac{1}{2} K A^2 \cos^2 \omega t$$

$$\boxed{KE = K_{max} \cos^2 \omega t}$$

Date

Potential Energy

$$\boxed{U = \frac{1}{2}Kx^2} = \frac{1}{2}KA^2\sin^2\omega t$$

$$= U_{max}\sin^2\omega t$$

$x = 0, U = 0$

(minimum)

$x = \pm A$

$U = \frac{1}{2}KA^2$

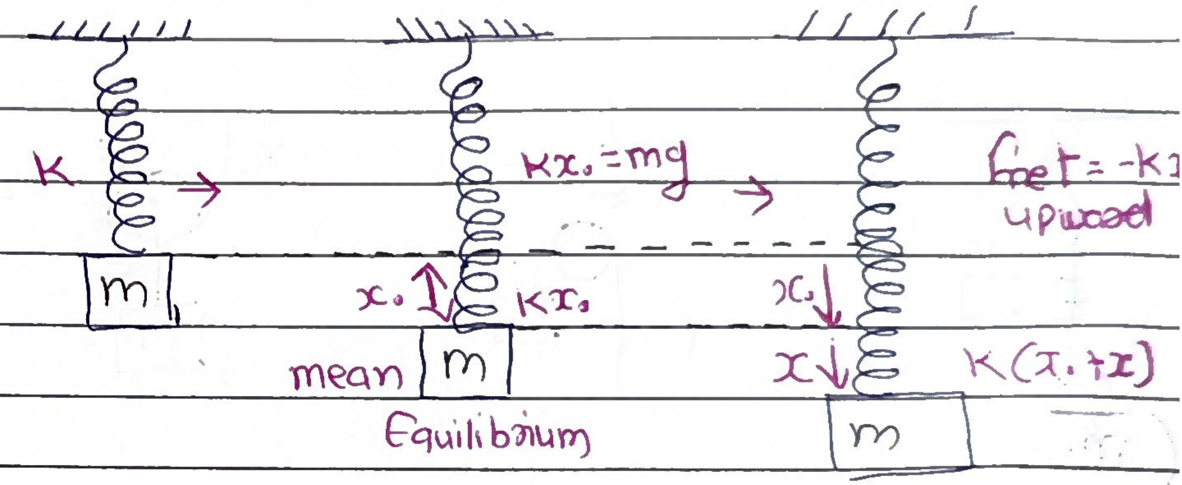
(maximum)

$$U_{max} = KE_{max} = \frac{1}{2}KA^2$$

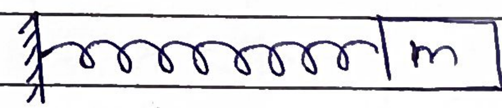
Date

Spring Mass System / oscillator

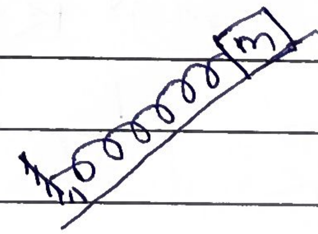
Case I



Case II



Case III

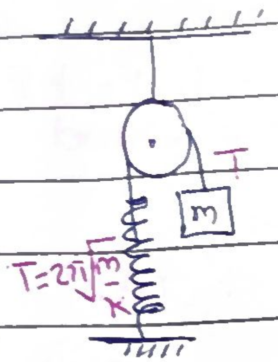


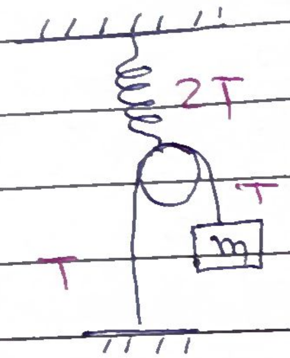
For case I II & III

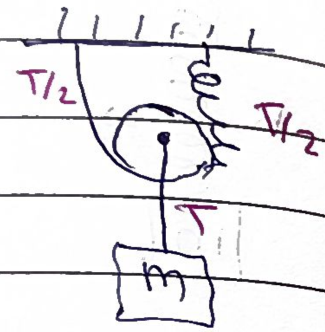
$$T = 2\pi\sqrt{\frac{m}{K}}$$

Date

Case IV :- mass is attached with string and string with springs







$K' = \left[\frac{T_m}{T_S}\right]^2 K$ | $K' = \left(\frac{T}{T}\right)^2 K$ | $K' = \left(\frac{T}{2T}\right)^2 K$

$K' = K$ | $K' = \frac{K}{4}$

T_m = mass in T | $T = 2\pi\sqrt{\frac{m}{K}}$ | $T = 2\pi\sqrt{\frac{4m}{K}}$

T_S = string in T

K' = effective

$$K' = \left(\frac{T}{T/2}\right)^2 K = K' = 4K$$

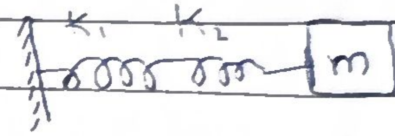
$$T = 2\pi\sqrt{\frac{m}{4K}}$$

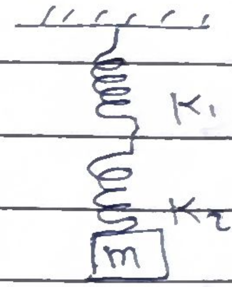
Date

Combination of Springs

1. Series Combination

Ends of both springs are joined with each othe



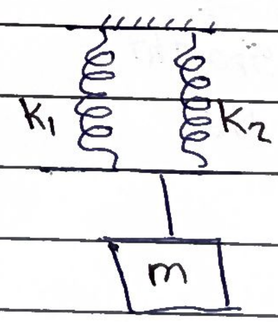


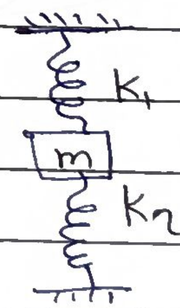
$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

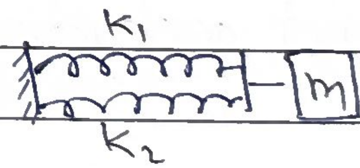
$$= \boxed{K_e = \frac{K_1 K_2}{K_1 + K_2}}$$

2. Parallel Combination

One end is joined with fixed end and other with mass



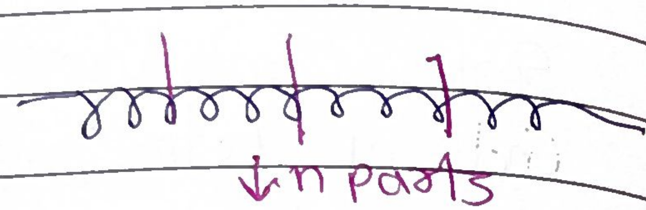




$$\boxed{K_e = K_1 + K_2}$$

Cutting of spring

$K \propto \frac{1}{l}$

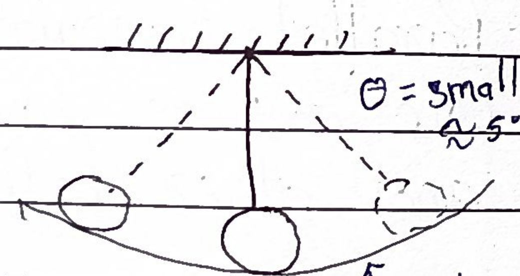


l/n

nK

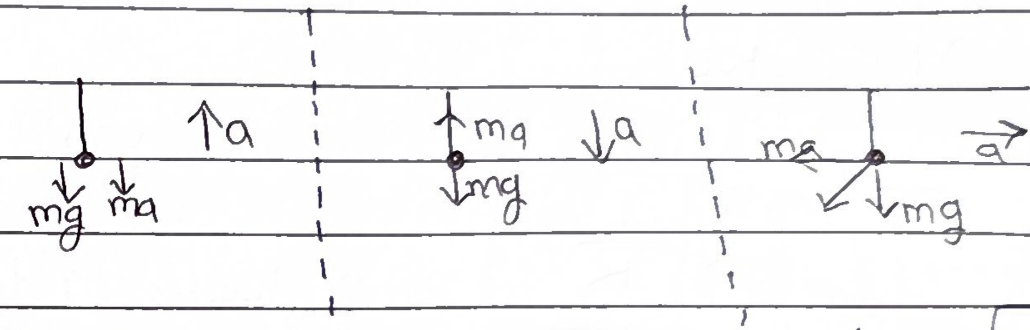
Simple Pendulum for small oscillations

$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

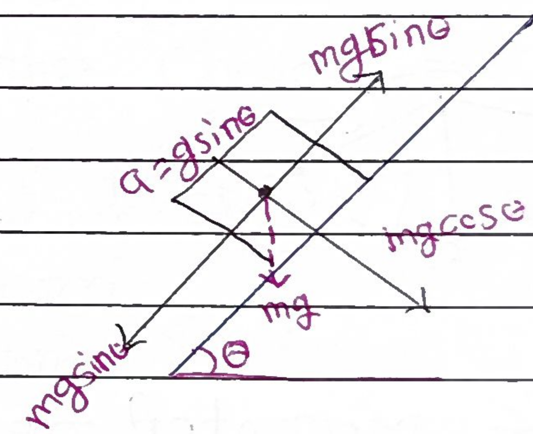


g_{eff} = net acceleration

Concept of g_{eff}



$g_{eff} = g + a$ $\qquad$ $g_{eff} = g - a$ $\qquad$ $g_{eff} = \sqrt{g^2 + a^2}$

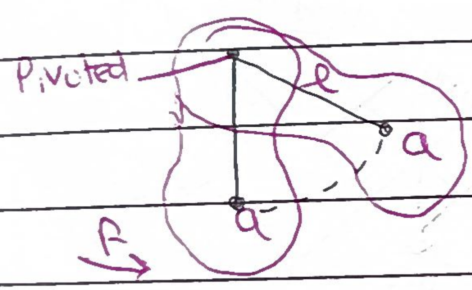


$g_{eff} = g\cos\theta$

5 Angular S.H.M

$$\boxed{\tau = -K\theta \qquad T = 2\pi\sqrt{\frac{I}{K}}}$$

Physical Pendulum → mass distributed



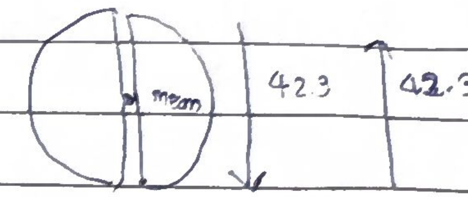
$T = 2\pi\sqrt{\frac{I}{mgl}}$ ← Moment of Inertia about Pivoted Point

l → distance b/w Centre Gravity & pivoted point

S.H.M of a body in a tunnel along any chord of Earth

1) Along diameter of Earth

S.H.M about Centre of earth



$T = 2\pi\sqrt{\frac{R_e}{g}} \approx 84.6$ mins

2) Any other Chord

SHM about centre of chord

T = same = 84.6 min

S

T=2π

K'

T_1

T

K

Oscillation of floating Body

Floating : equilibrium

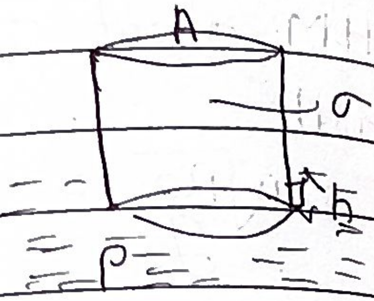
Slightly displace → S.H.M

A

σ

h

ρ



$$\boxed{T = 2\pi\sqrt{\frac{m}{A\rho g}}}$$

m = mass of body

ρ = density of fluid

A = cross section area

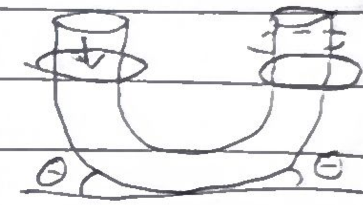
$$T = 2\pi\sqrt{\frac{\sigma A L}{A\rho g}} = 2\pi\sqrt{\frac{\sigma L}{\rho g}}$$

$$T = 2\pi\sqrt{\frac{\rho h}{\rho g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{h}{g}}}$$

Oscillation of liquid Column

$$T = 2\pi\sqrt{\frac{m}{A\rho g(\sin\theta_1 + \sin\theta_2)}}$$



θ θ

For U-tube, $\theta_1 = \theta_2 = 90°$

$$T = 2\pi\sqrt{\frac{m}{2A\rho g}}$$

* Combination of two or more S.H.M

1. In same direction

$x_1 = A_1 \sin\omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

$x_R = x_1 + x_2$ (SHM)

$$\boxed{x_R = A_R \sin(\omega t + \theta)}$$

Date

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

$$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Note =

$$\cos\omega t = \sin(\omega t + 90^\circ)$$

$$-\cos\omega t = \sin(\omega t - 90^\circ)$$